6. Centrically loaded glass column

Assess centrically loaded glass column, see Fig. 6.1. Centrical force $\mathbf{F_{Ed}}$ which involves a load resulting from self-weight of the column, roof and snow. The column is made of tripple layered thermally toughened glass **3x 12 mm** bonded with PVB foil **2x 0,76 mm.** The column´s width is **b = 400 mm**, its height is **L = 3,0m** and is simply hinged on both ends.

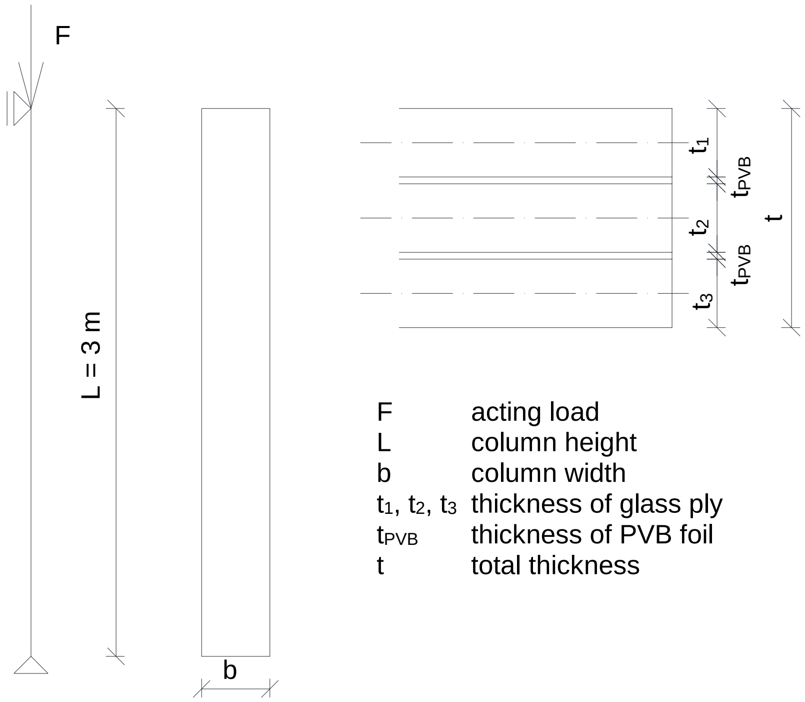

Fig. 6.1: Construction schema

6.1 Loading

Load actions

Column is loaded with a centric force $\mathbf{F_k = G_k + Q_k}$. Characteristic value of permanent load is $\mathbf{G_k = 2000\ N}$. Characteristic value of variable snow load is $\mathbf{Q_k = 1000\ N}$.

Load combinations

There are three load combinations considered for our column assessment. Combination **KZ1** is considered for ultimate limit state assessment, combination **KZ2** is considered for accidental situation assessment (when one glass ply is broken). In seviceability limit state (column´s horizontal deflections), there is combination **KZ3** taken into account.

Combination **KZ1** according to EN 1990, [1]

$$F_{Ed,1} = \gamma_G \cdot G_k + \gamma_Q \cdot Q_k = 1{,}35 \cdot 2000 + 1{,}5 \cdot 1000 = 4200\ N = 4{,}2\ kN$$

Combination **KZ2** according to [1]

$$F_{Ed,2} = \gamma_G \cdot G_k + \psi_1 \cdot \gamma_Q \cdot Q_k = 1{,}0 \cdot 2000 + 0{,}5 \cdot 1{,}0 \cdot 1000 = 2500\ N = 2{,}5\ kN$$

Combination **KZ3** according to [1]

$$F_k = \gamma_G \cdot G_k + \gamma_Q \cdot Q_k = 1{,}0 \cdot 2000 + 1{,}0 \cdot 1000 = 3000\ N = 3{,}0\ kN$$

6.2 Cross-section characteristics

For ULS and SLS glass column assessment, it is desirable to determine on glass ply cross-section characteristics, see Fig. 6.2.

$$t_1 = t_2 = t_3 = 12\ mm$$

Cross-section area

$$A = b \cdot t_1 = 400 \cdot 12 = 4800\ mm^2$$

Moment of inertia

$$I_y = \frac{b \cdot {t_1}^3}{12} = \frac{400 \cdot 12^3}{12} = 57600\ mm^4$$

Cross-section modulus

$$W_y = \frac{b \cdot {t_1}^2}{6} = \frac{400 \cdot 12^2}{6} = 9600\ mm^3$$

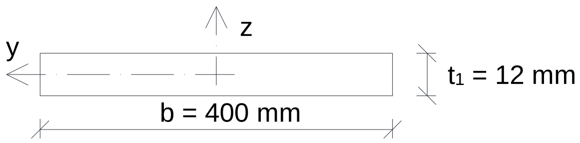

Fig. 6.2: Assessed cross-section schema

6.3 Internal forces determination

Normal forces

Laminated glass column has to be assessed without glass plies shear bonding effect according to DIN 18008-1, [2]. The applied load is divided between the individual plies proportionally to their bending stiffness.

For glass plies of the same thickness $\boldsymbol{t_1}$ **= 12 mm** can be assumed:

$$N_{Ed} = \frac{F_{Ed}}{n}$$

where N_{Ed} is normal force in one ply;

F_{Ed} value of the acting load;

n number of plies in considered cross-section.

By substitution into relation for the individual combinations we obtain:

Combination **KZ1** – ***n* = 3**

$$N_{Ed,1} = \frac{F_{Ed,1}}{n} = \frac{4200}{3} = 1400\ N = 1,4\ kN$$

Combination **KZ2** – ***n* = 2** (accidental situation – one of the plies is broken)

$$N_{Ed,2} = \frac{F_{Ed,2}}{n} = \frac{2500}{2} = 1250\ N = 1,25\ kN$$

Combination **KZ3** – ***n* = 3**

$$N_k = \frac{F_k}{n} = \frac{3000}{3} = 1000\ N = 1,0\ kN$$

Bending moments

Internal forces are determined for the imperfect column with respect to the safe design. In spite of the fact that the load is centrically applied, there is the arising bending moment resulting from the imperfection. This bending moment causes the additional horizontal deflection and the growth of the additional internal bending moment till column´s colapse (2nd order effect).

The initial imperfection value for thermally modified glass:

$$w_0 = \frac{L}{300} = \frac{3000}{300} = 10\ mm$$

where w_0 is initial imperfection;

L column length.

In our case, there is Euler´s critical force only for one individual glass ply considered:

$$N_{cr} = \frac{\pi^2 \cdot E \cdot I_y}{L_{cr}^2} = \frac{\pi^2 \cdot 70 \cdot 10^3 \cdot 57600}{3000^2} = 4421\ N = 4{,}42\ kN$$

where E is Young´s modulus of glass;

I_y moment of inertia.

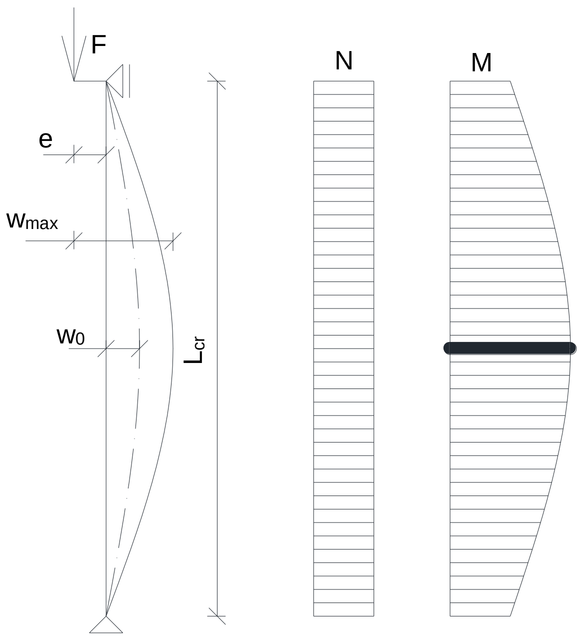

Fig. 6.3: Imperfect column internal forces

The ultimate horizontal deflection w_{max} can be obtained from differential 2nd order equation solution for the imperfect column in compression, see Fig. 6.3:

$$w_{max} = \frac{e}{cos\left(\frac{L_{cr}}{2} \cdot \left(\frac{N_{Ed}}{N_{cr}}\right)^{-0,5}\right)} + \frac{w_0}{1 - \frac{N_{Ed}}{N_{cr}}}$$

where e is acting force eccentricity;

L_{cr} buckling length;

N_{cr} Euler´s critical normal force.

By substitution into relations we obtain horizontal deflection:

For combination **KZ1**:

$$w_{max,1} = \frac{e}{cos\left(\frac{L_{cr}}{2}\cdot\left(\frac{N_{Ed,1}}{N_{cr}}\right)^{-0,5}\right)} + \frac{w_0}{1-\frac{N_{Ed,1}}{N_{cr}}} = \frac{0,0}{cos\left(\frac{3000}{2}\cdot\left(\frac{1400}{4421}\right)^{-0,5}\right)} + \frac{10,0}{1-\frac{1400}{4421}} = 14,6\ mm$$

For combination **KZ2**:

In this combination there is the eccentricity **e = 6,0 mm** resulting from the outer glass ply breakage.

$$w_{max,2} = \frac{e}{cos\left(\frac{L_{cr}}{2}\cdot\left(\frac{N_{Ed,2}}{N_{cr}}\right)^{-0,5}\right)} + \frac{w_0}{1-\frac{N_{Ed,2}}{N_{cr}}} = \frac{6,0}{cos\left(\frac{3000}{2}\cdot\left(\frac{1250}{4421}\right)^{-0,5}\right)} + \frac{10,0}{1-\frac{1250}{4421}} = 19,9\ mm$$

Bending moment is determined as:

$$M_{Ed} = N_{Ed}\cdot w_{max}$$

where M_{Ed} is design bending moment;

w_{max} total horizontal deflection.

By substitution into relations we obtain bending moments:

For combination **KZ1**:

$$M_{Ed,1} = N_{Ed,1}\cdot w_{max,1} = 1400\cdot 14,6 = 20440\ Nmm$$

For combination **KZ2**:

$$M_{Ed,2} = N_{Ed,2}\cdot w_{max,2} = 1250\cdot 19,9 = 24875\ kNm$$

Final internal forces

Výsl Final internal forces acting in one glass ply of the column are summarized in Tab. 6.1.

Tab. 6.1: Final internal forces

Combinations	N_{Ed} [kN]	e [mm]	w_0 [mm]	N_{cr} [kN]	w_{max} [mm]	M_{Ed} [kNm]
KZ1	1,4	0,0	10	4,42	14,6	0,02044
KZ2	1,25	6,0	10	4,42	19,9	0,02488

6.4 Material properties

Minimum characterictic tensile strenght of thermally toughened glass in bending is $\boldsymbol{f_{b,k}}$ **= 120 N/mm²** according to EN 12150-1, [3]. Young´s modulus of elasticityis ***E* = 70 GPa** according to EN 572-1, [4].

Nign tensile strenght of tempered glass according to DIN 18008-1, [2] is determined as:

$$f_{b,d} = \frac{k_c\cdot f_{b,k}}{\gamma_M} = \frac{1,0\cdot 120}{1,5} = 80\ MPa$$

where $f_{b,d}$ is design value of thermally toughened glass strength;

$f_{b,k}$ characteristic value of thermally toughened glass strength;

k_c construction factor ($k_c = 1,0$);

γ_M partial factor of material (pro tepelně upravené sklo $\gamma_M = 1,5$).

6.5 Ultimate limit state

For the ultimate limit state assessment, the positive glass plies shear coupling will be neglected. The calculation is therefore safe and conservative. The maximum stress in one glass ply for the normal force acting together with bending moment is determined as:

$$\sigma_{t,d} = \frac{N_{Ed}}{A} \pm \frac{M_{Ed}}{W_y}$$

where $\sigma_{t,d}$ is tensile stress in the cross-section;

A cross-section area;

W_y cross-section modulus;

$f_{b,d}$ design value of thermally toughened glassstrength.

By substitution into relation for the individual combinations we get values for individual combination.

For combination **KZ1**:

$$\sigma_{t,d1} = \frac{N_{Ed,1}}{A} + \frac{M_{Ed,1}}{W_y} = -\frac{1400}{4800} + \frac{20440}{9600} = 1{,}84\ MPa$$

For combination **KZ2**:

$$\sigma_{t,d2} = \frac{N_{Ed,2}}{A} + \frac{M_{Ed,2}}{W_y} = -\frac{1250}{4800} + \frac{24875}{9600} = 2{,}33\ MPa$$

Stress assessment:

$$\sigma_{t,d1} = 1{,}84\ MPa \ \leq 80\ MPa = f_{b,d} \ \Rightarrow \ SATISFACTORY$$

$$\sigma_{t,d2} = 2{,}33\ MPa \ \leq 80\ MPa = f_{b,d} \ \Rightarrow \ SATISFACTORY$$

Stability assessment:
The loss of compressed member stability can be simply assessed according to the relation:

$$\frac{N_{Ed}}{\frac{N_{cr}}{\gamma}} + \frac{M_{Ed}}{M_{Rd}} \leq 1{,}0$$

where γ is safety factor (γ = 1,7);

M_{Rd} design bending resistance obtained from the relation:

$$M_{Rd} = f_{b,d} \cdot W_y = 80 \cdot 9600 = 768000\ Nmm = 0{,}768\ kNm$$

For combination **KZ1**:

$$\frac{N_{Ed,1}}{\frac{N_{cr}}{\gamma}} + \frac{M_{Ed,1}}{M_{Rd}} = \frac{1400}{\frac{4421}{1{,}7}} + \frac{20440}{768000} = 0{,}565 \leq 1{,}0 \ \Rightarrow \ SATISFACTORY$$

For combination **KZ2**:

$$\frac{N_{Ed,2}}{\frac{N_{cr}}{\gamma}} + \frac{M_{Ed,2}}{M_{Rd}} = \frac{1250}{\frac{4421}{1{,}7}} + \frac{24875}{768000} = 0{,}513 \leq 1{,}0 \ \Rightarrow \ SATISFACTORY$$

6.6 Serviceability limit state

There is no positive glass plies shear coupling effect considered in SLS. The horizontal column´s deflection in the middle of its height δ_{max} is the assessed value. This can be calculated as:

$$\delta_{max} = \frac{w_0}{1 - \frac{N_k}{N_{cr}}} - w_0 = \frac{10}{1 - \frac{1000}{4421}} - 10 = 2{,}92\ mm$$

The limit value of deflection δ_{lim} was chosen as 1/300 of the span in according to the reccomended values of horizontal deflections of steel columns and wands asglazed walls parts. These may by found in EN 1993-1-1, [5].

Assessment:

$$\delta_{max} = 2{,}92\ mm\ \leq \delta_{lim} = \frac{L}{300} = \frac{3000}{300} = 10{,}00\ mm\ \Rightarrow\ SATISFACTORY$$

6.7 Literature

[1] ČSN EN 1990, *Eurokód: Zásady navrhování konstrukcí*, 2015. Ed. 2. Úřad pro technickou normalizaci, metrologii a státní zkušebnictví

[2] DIN 18008-1, *Glas im Bauwesen – Bemessungs- und Konstruktionsregeln: Teil 1: Begriffe und allgemeine Grundlagen*, 2010. Berlin: Deutsches Institut für Normun

[3] ČSN EN 12150-1, *Sklo ve stavebnictví – Tepelně tvrzené sodnovápenatokřemičité bezpečnostní sklo – Část 1: Definice a popis*, 2016. Úřad pro technickou normalizaci, metrologii a státní zkušebnictví

[4] ČSN EN 572-1, *Sklo ve stavebnictví – Základní výrobky ze sodnovápenatokřemičitého skla – Část 1: Definice a obecné fyzikální a mechanické vlastnosti*, 2012. Úřad pro technickou normalizaci, metrologii a státní zkušebnictví

[5] ČSN EN 1993-1-1, *Eurokód 3: Navrhování ocelových konstrukcí – Část 1-1: Obecná pravidla a pravidla pro pozemní stavby*, 2006. Úřad pro technickou normalizaci, metrologii a státní zkušebnictví