2. Rectangular double-pane insulating glass unit

Check facade panel made of insulating double-pane glazing with total dimensions of **1,5 x 2 m** is designed. The double glazing is assumed as simply circumferentially supported. Inner and outer float glass panes having the same thickness **7 mm** were used in the structure, the width between them was chosen as **16 mm**, see Fig. 2.2.

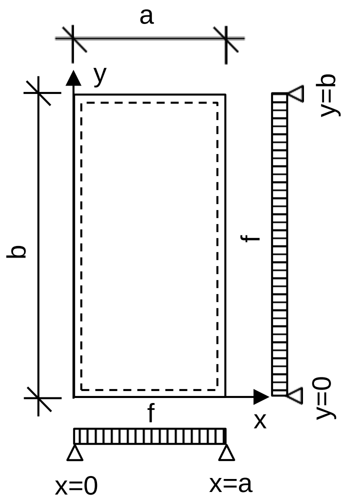


Fig. 2.1: Struction scheme

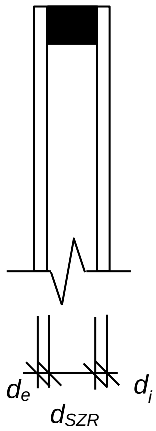


Fig. 2.2: Double pane cross-section

Double pane length	a = 1500 mm
Double pane height	b = 2000 mm
Inner pane thickness	d_i = 7 mm
Outer pane thickness	d_e = 7 mm
Gap width	d_{SZR} = 16 mm

2.1 Loading

Wind loading of our considered façade is determined according to EN 1991-1-4 [1]. Internal loading specific for insulating glass is determined according to german standarts DIN 18008-1 [2] and DIN 18008-2 [3]. In case of double insulating glass, it becomes neccessary to consider summer and winter period when the main loading cases are temperature change ΔT, atmospheric pressure change Δp_{met} and height above the sea level change, see Tab. 2.1.

Tab. 2.1: Load combination for insulating glass

Season	**Temperature change ΔT [K]**	**Atm. pressure change Δp_{met} [kN/m²]**	**Altitude change ΔH [m]**
Summer	+ 20	- 2,0	+ 600
Winter	- 25	+ 4,0	- 300
Values in this table are appropriate to the basic calculation. If higher changes in the atmosferic pressure, temperature or the height above the sea are expected, it is neccessary to consider real values. On the other hand, when the differences are demonstrably lower, it becomes neccessary to work out the assessment with lower values.			

Altitude change:

The load calculation due to the changes in the altitude shall be carried out in accordance with DIN 18008-2, [3] as:

$$\Delta p_{geo,summer} = 0{,}012\ \frac{kN}{m^3} \cdot \Delta H = 0{,}012 \cdot (+\,600) = 7{,}2\ \frac{kN}{m^2}$$

$$\Delta p_{geo,winter} = 0{,}012\ \frac{kN}{m^3} \cdot \Delta H = 0{,}012 \cdot (-\,300) = -3{,}6\ \frac{kN}{m^2}$$

where Δp_{geo} is plane load due to changes in altitude;

ΔH difference in altitude in [m] (see Tab. 2.1).

Change of the atmospheric pressure due to changes in meteorological conditions:

The calculation of the load resulting from the change of the pressure and temperature shall be carried out in accordance with DIN 18008-2, [3] as:

$$\Delta p_{summer} = \Delta p_{\Delta T} - \Delta p_{met} = 0{,}34\ \frac{kN}{K \cdot m^2} \cdot \Delta T - \Delta p_{met} = 0{,}34 \cdot (+\,20) - (-\,2) = 8{,}8\ \frac{kN}{m^2}$$

$$\Delta p_{winter} = \Delta p_{\Delta T} - \Delta p_{met} = 0{,}34\ \frac{kN}{K \cdot m^2} \cdot \Delta T - \Delta p_{met} = 0{,}34 \cdot (-\,25) - (+\,4) = -12{,}5\ \frac{kN}{m^2}$$

where Δp is plane load due to changes in pressure and temperature;

$\Delta p_{\Delta T}$ plane load due to temperature changes;

Δp_{met} plane load due to pressure changes.

Wind load:

The calculation is executed for the wind area II. and terrain category IV according to [1]. Characteristic wind velocity for this area is $\boldsymbol{v_b}$ **= 25 m/s,** characteristic middle wind velocity 6 m above the ground is $\boldsymbol{v_m}$***(6 m)*** **= 13,5 m/s**. Characteristic peak dynamic pressure appropriate to the aforementioned values is $\mathbf{q_p}$**(6 m) = 0,46 kN/m²**. The external pressure coefficient for the windward face is (with solidity ratio φ = 1,0) $\mathbf{c_{pe,10,pressure}}$ **= 0,8**. The external pressure coefficient for the leeward face is $\mathbf{c_{pe,10,suction}}$ **= - 0,7**. Wind load perpendicular to the facade is therefore considered with its characteristic value of the pressure $\mathbf{w_{k,pressure}}$ **= 0,37 kN/m²** and of the suction $\mathbf{w_{k,suction}}$ **= - 0,32 kN/m²**.

Self weight of the double glazing is perpendicular to the prevailing load, it can therefore be neglected.

2.2 Conversion into individual panes

In the case of insulating glass, the coefficient *φ* is used to combine the individual load cases, whereby the applied load is redistributed to the individual boards. The following procedure for determining the coefficient of insulating glass is given in Annex A of DIN 18008-2, [3].

Tab. 2.2: Table for coefficient B_v calculation

a/b	1,0	0,9	0,8	0,7	0,6	0,5	0,4	0,3	0,2	0,1
B_v	0,0194	0,0237	0,0288	0,0350	0,0421	0,0501	0,0587	0,0676	0,0767	0,0857
Linear interpolation can be used for intermediate values. For parameter *a*, a smaller dimension of the board is taken, *b* is the remaining dimension.										

Calculation of B_v:

$$\frac{a}{b} = \frac{1,5}{2} = 0,75 \Rightarrow B_v = 0,0319$$

The characteristic edge length a^* is determined in the case of insulating double glazing as:

$$a^* = 28,9 \cdot \sqrt[4]{\frac{d_{SZR} \cdot {d_i}^3 \cdot {d_e}^3}{({d_i}^3 + {d_e}^3) \cdot B_v}} = 28,9 \cdot \sqrt[4]{\frac{16 \cdot 7^3 \cdot 7^3}{(7^3 + 7^3) \cdot 0,0319}} = 494,93\ mm$$

where	a^*	is	characteristic edge length;
	d_{SZR}		distance between the panes;
	d_i		thickness of the inner glass pane;
	d_e		thickness of the outer glass pane;
	B_V		factor (see Tab. 2.2).

The insulating glass factor is then determined as:

$$\varphi = \frac{1}{1 + \left(\frac{a}{a^*}\right)^4} = \frac{1}{1 + \left(\frac{1500}{494,93}\right)^4} = 0,012$$

where	φ	is	insulating glass factor;
	a		smaller size of the plate dimensions (width *a* x height *b*).

In the case of the wind load, the stiffness ratio of the individual panes also affects the load distribution, which is taken into account by the stiffness factors δ_e for the outer pane and δ_i for the inner pane, which are determined as:

$$\delta_e = \frac{{d_e}^3}{{d_i}^3 + {d_e}^3} = \frac{7^3}{7^3 + 7^3} = 0,5$$

$$\delta_i = 1 - \delta_e = 1 - 0,5 = 0,5$$

where	δ_e	is	stiffness factor of the outer pane;
	δ_i		stiffness factor of the inner pane.

If the outer and inner panes are of the same thickness, is $\delta_e = \delta_i$, i.e. in this case $\delta_e = \delta_i =$ **0,5.**

Tab. 2.3: Distribution of the actions

Load acting on	**Action**	**Proportion of load acting on**	
		outer pane	**inner pane**
Outer	Wind w_e	$(\delta_e + \varphi \cdot \delta_i) \cdot w_e$	$(1 - \varphi) \cdot \delta_i \cdot w_e$
	Snow s	$(\delta_e + \varphi \cdot \delta_i) \cdot s$	$(1 - \varphi) \cdot \delta_i \cdot s$
Inner	Wind w_i	$(1 - \varphi) \cdot \delta_e \cdot w_i$	$(\delta_i + \varphi \cdot \delta_e) \cdot w_i$
Both	Isochoric pressure	$- \varphi \cdot p_0$	$+ \varphi \cdot p_0$
The isochoric pressure includes the load resulting from the altitude change, pressure change and temperature change.			

Load cases:

ZS1 – influence of altitude change (in summer)

outer pane loading: $\mathrm{f}_{1,\mathrm{e}} = -\,\varphi \cdot \Delta \mathrm{p}_{\mathrm{geo,summer}} = -0{,}012 \cdot 7{,}2 = -0{,}08\ kN/m^2$

inner pane loading: $f_{1,i} = \varphi \cdot \Delta p_{geo,summer} = 0{,}012 \cdot 7{,}2 = 0{,}08\ kN/m^2$

ZS2 – influence of atmospheric pressure and temperature change (in summer)

outer pane loading: $\mathrm{f}_{2,\mathrm{e}} = -\,\varphi \cdot \Delta \mathrm{p}_{\mathrm{summer}} = -0{,}012 \cdot 8{,}8 = -0{,}10\ kN/m^2$

inner pane loading: $f_{2,i} = \varphi \cdot \Delta p_{summer} = 0{,}012 \cdot 8{,}8 = 0{,}10\ kN/m^2$

ZS3 – influence of altitude change (in winter)

outer pane loading: $f_{3,e} = -\,\varphi \cdot \Delta p_{geo,winter} = -0{,}012 \cdot (-3{,}6) = 0{,}04\ kN/m^2$

inner pane loading: $f_{3,i} = \varphi \cdot \Delta p_{geo,winter} = 0{,}012 \cdot (-3{,}6) = -0{,}04\ kN/m^2$

ZS4 – influence of atmospheric pressure and temperature change (in winter)

outer pane loading: $f_{4,e} = -\,\varphi \cdot \Delta p_{winter} = -0{,}012 \cdot (-12{,}5) = 0{,}15\ kN/m^2$

inner pane loading: $f_{4,i} = \varphi \cdot \Delta p_{winter} = 0{,}02 \cdot (-12{,}5) = -0{,}15\ kN/m^2$

ZS5 – wind pressure

outer pane loading: $f_{5,e} = (\delta_e + \varphi \cdot \delta_i) \cdot \mathrm{w}_{\mathrm{k,pressure}} = (0{,}5 + 0{,}012 \cdot 0{,}5) \cdot 0{,}37 = 0{,}19\ kN/m^2$

inner pane loading: $f_{5,i} = (1 - \varphi) \cdot \delta_i \cdot \mathrm{w}_{\mathrm{k,pressure}} = (1 - 0{,}012) \cdot 0{,}5 \cdot 0{,}37 = 0{,}18\ kN/m^2$

ZS6 – wind suction

outer pane loading: $f_{6,e} = (\delta_e + \varphi \cdot \delta_i) \cdot \mathrm{w}_{\mathrm{k,suction}} = (0{,}5 + 0{,}012 \cdot 0{,}5) \cdot (-0{,}32)$

$f_{6,e} = -0{,}16\ kN/m^2$

inner pane loading: $f_{6,i} = (1 - \varphi) \cdot \delta_i \cdot \mathrm{w}_{\mathrm{k,suction}} = (1 - 0{,}012) \cdot 0{,}5 \cdot (-0{,}32)$

$\mathrm{f}_{6,\mathrm{e}} = -0{,}16\ \mathrm{kN/m^2}$

Load combinations:

When combining load cases, it is important to combine only those cases causing the same stresses on the solved pane. For example, when assessing the external pane, the load resulting from the wind pressure and the underpressure load will be combined, since both loads attract the inner glass surface by tension and so on, see Fig. 2.3.

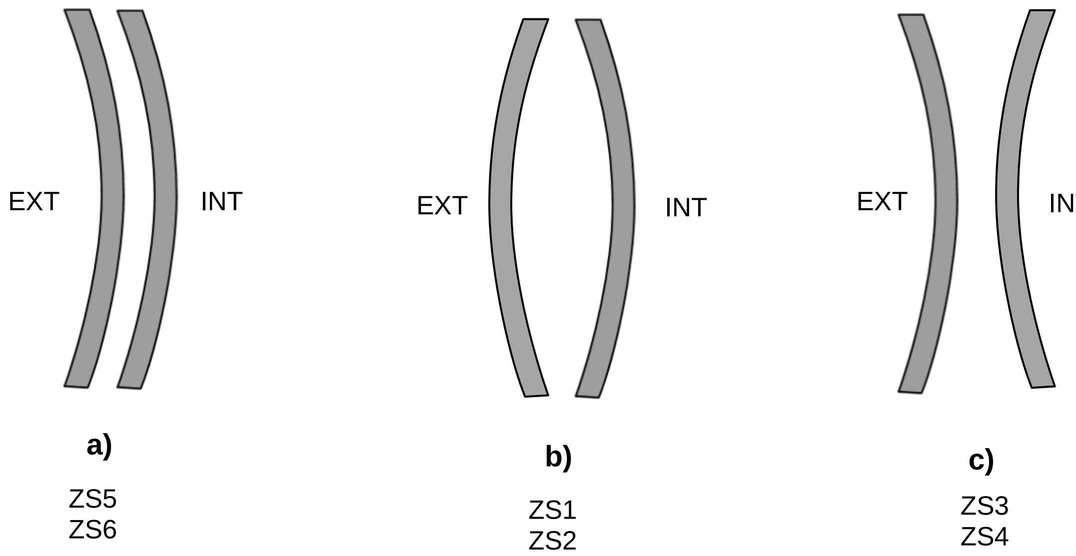


Fig. 2.3: Stress of the insulating double glazing resulting from the external load (a), overpressure (b) and lower pressure (c)

The combinations listed in Tab. 2.5 are based on Equation 6.10, Article 6.4.3.2 (3) of the EN 1990 standard, [4]. The modification coefficient k_{mod} considering the impact of the load duration length (see Tab. 2.4) is determined in accordance with the Article 8.3.7 of DIN 18008-1, [2] for each combination separately. The load with the shortest duration is crucial.

Tab. 2.4: Modification coefficient values

Load duration	**Example**	k_{mod}
Permanent	Self weight, altitude change	0,25
Medium-term	Snow, isochoric change	0,4
Instantaneous	Wind, impact	0,7

Since both glass panes having the same thickness are used, more stressed pane can be only considered. In this example, the outer pane is more stressed due to the slightly higher wind pressure acting on it, therefore the load capacity verification will be carried out for the outer one.

Tab. 2.5: Load combinations

Nr.	**Designation**	**Load duration**	k_{mod} **[-]**	**Value** $f_{e,d}$ **[kN/m²]**	**Ratio** $f_{e,d}/k_{mod}$ **[kN/m²]**	**Limit state**
KZ1	$\gamma_G \cdot ZS1$	Permanent	0,25	- 0,11	- 0,46	ULS
KZ2	$\gamma_G \cdot ZS1 + \gamma_Q \cdot ZS2$	Medium-term	0,4	- 0,27	- 0,67	
KZ3	$\gamma_G \cdot ZS1 + \gamma_Q \cdot ZS6 + \gamma_Q \cdot \psi_0 \cdot ZS2$	Instantaneous	0,7	- 0,45	- 0,64	
KZ4	$\gamma_G \cdot ZS1 + \gamma_Q \cdot ZS2 + \gamma_Q \cdot \psi_0 \cdot ZS6$	Instantaneous	0,7	- 0,42	- 0,59	
KZ5	$\gamma_G \cdot ZS3$	Permanent	0,25	+ 0,06	+ 0,23	
KZ6	$\gamma_G \cdot ZS3 + \gamma_Q \cdot ZS4$	Medium-term	0,4	+ 0,28	**+ 0,69**	

KZ7	$\gamma_G \cdot ZS3 + \gamma_Q \cdot ZS5 + \gamma_Q \cdot \psi_0 \cdot ZS4$	Instantaneous	0,7	+ 0,47	+ 0,67	
KZ8	$\gamma_G \cdot ZS3 + \gamma_Q \cdot ZS4 + \gamma_Q \cdot \psi_0 \cdot ZS5$	Instantaneous	0,7	+ 0,44	+ 0,63	
KZ9	$\gamma_G \cdot ZS1 + \gamma_Q \cdot ZS2 + \gamma_Q \cdot \psi_0 \cdot ZS6$	-	-	+ 0,29	-	SLS
KZ10	$\gamma_G \cdot ZS1 + \gamma_Q \cdot ZS6 + \gamma_Q \cdot \psi_0 \cdot ZS2$	-	-	+ 0,31	-	
KZ11	$\gamma_G \cdot ZS3 + \gamma_Q \cdot ZS4 + \gamma_Q \cdot \psi_0 \cdot ZS5$	-	-	+ 0,30	-	
KZ12	$\gamma_G \cdot ZS3 + \gamma_Q \cdot ZS5 + \gamma_Q \cdot \psi_0 \cdot ZS4$	-	-	**+ 0,32**	-	

Combination factor for permanent loading in ULS is ψ_G = **1,35** and for variable loading in ULS is ψ_Q = **1,5.** Reduction combination factor ψ_0 is considered according to DIN 18008-1, [2] as **0,6**.

Crucial loading combination in ULS is given by the highest ratio of f_e and modification factor k_{mod}. In our case, combination KZ6 expressing height above the sea level change, atmospheric pressure change and temperature change in a winter is the crucial one. The crucial combination for SLS is given by the highest load value of loading f_e, i.e. KZ12 expressing wind loading, height above the sea level change, atmospheric pressure change and temperature change in a winter.

2.3 Material characteristics of glass

Float glass of the thickness $\boldsymbol{d_i = d_e = 7}$ **mm** was chosen for the design. The minimum characteristic strength of tempered soda-lime glass is according to EN 572-1, [5] $\boldsymbol{f_{g,k} = 45}$ **N/mm²** and Young's modulus of elasticity is $\boldsymbol{E = 70}$ **GPa**.

The glass design strength determination was established according to the procedure in DIN 18008-1, [2]:

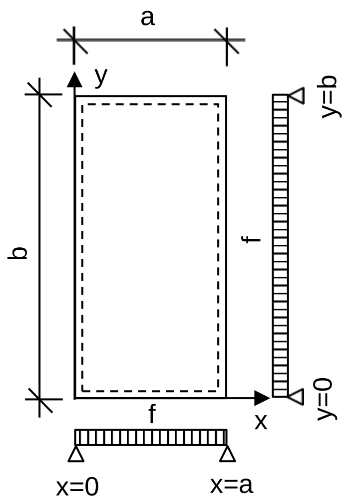
$$f_{g,d} = \frac{k_{mod} \cdot k_c \cdot f_{g,k}}{\gamma_M} = \frac{1{,}8 \cdot 0{,}4 \cdot 45}{1{,}8} = 18\ MPa$$

where	$f_{g,d}$	is	design bending strength;
	$f_{g,k}$		characteristic bending strength;
	γ_M		partial factors for material properties (for float glass $\gamma_M = 1{,}8$);
	k_{mod}		modification factor (see Tab. 2.4);
	k_c		construction factor (for float glass $k_c = 1{,}8$).

2.4 Stress calculation

The calculation of the pane stress can be performed in many ways. In this example, the empirical method is described.

Tab. 2.6: Values of the pane bending moments calculation



a – smaller size of the pane

ν = 0,23

The maximum values are in the middle of the pane

$$m_x = f_d \cdot k_x \cdot a$$

$$m_y = f_d \cdot k_y \cdot a$$

The maximum deflection

$$\delta = k_w \cdot \frac{f_d \cdot a^4}{E \cdot \mathrm{d}^3}$$

a/b	k_w	k_x	k_y
0,50	0,115167	0,097289	0,026449
0,55	0,106643	0,090649	0,026816
0,60	0,098309	0,084101	0,027583
0,65	0,090164	0,077822	0,028696
0,70	0,082492	0,071912	0,030178
0,75	0,075294	0,066294	0,032106
0,80	0,06857	0,061045	0,034403
0,85	0,062414	0,056296	0,036815
0,90	0,056637	0,052047	0,039342
0,95	0,051238	0,048344	0,042184
1,00	0,046124	0,045264	0,045264

The values were obtained from the tables [6] considering the Poisson´s coefficient ν = 0,23 of the corresponding glass. Intermediate values can be obtained by linear interpolation.

The board bending moments calculation is performed according to:

$$m_x = k_x \cdot f_e \cdot a^2 = 0{,}066294 \cdot 0{,}28 \cdot 1{,}5^2 = 0{,}042\ kNm/m´$$

$$m_y = k_y \cdot f_e \cdot b^2 = 0{,}032106 \cdot 0{,}28 \cdot 2^2 = 0{,}036\ kNm/m´$$

where m_x is bending moment in x-direction;

m_y bending moment in y-direction;

k_x, k_y factors for the moments calculation (see Tab. 2.6);

a smaller size of the pane´s dimensions [m];

b larger size of the pane´s dimensions [m];

f_e design load.

From the bending moments, it is then possible to calculate the stress in the direction of the axis x σ_x, in the direction of the axis y σ_y and the shear stress τ_{xy}. In the case of this rectangular pane loaded in this way, the torque is zero at the point of the maximum bending moments (i.e., in the middle of the pane). Thus the shear stress will be zero:

$$\sigma_x = \frac{12 \cdot m_x}{{d_e}^3} \cdot \frac{d_e}{2} = \frac{12 \cdot 0{,}042}{(7 \cdot 10^{-3})^3} \cdot \frac{7 \cdot 10^{-3}}{2} = 5{,}14 \cdot 10^3\ kPa = 5{,}14\ MPa$$

$$\sigma_y = \frac{12 \cdot m_y}{{d_e}^3} \cdot \frac{d_e}{2} = \frac{12 \cdot 0{,}036}{(7 \cdot 10^{-3})^3} \cdot \frac{7 \cdot 10^{-3}}{2} = 4{,}41 \cdot 10^3\ kPa = 4{,}41\ MPa$$

$$\tau_{xy} = \frac{12 \cdot m_{xy}}{{d_e}^3} \cdot \frac{d_e}{2} = 0\ MPa$$

where σ_x is stress in the x-direction;
σ_y stress in the y-direction;
τ_{xy} shear stress;
d_e pane thickness [m].

Since the shear stress τ_{xy} is zero, the main stresses σ_1 and σ_2 are identical to the stresses σ_x and σ_y:

$$\sigma_1 = \sigma_x = 5{,}14\ MPa$$

$$\sigma_2 = \sigma_y = 4{,}41\ MPa$$

where σ_1, σ_2 are main stresses.

2.5 Ultimate limit state

KZ6 assessment:

$$\sigma_1 = 5{,}14\ MPa \le 18\ MPa = f_{g,d} \Rightarrow SATISFACTORY$$

$$\sigma_2 = 4{,}41\ MPa \le 18\ MPa = f_{g,d} \Rightarrow SATISFACTORY$$

The pane is at ULS **satisfactory** for the applied load.

2.6 Serviceability limit state

In the SLS, the deflection will be investigated. The limit deflection of the simply supported panes is *a*/100, where *a* is smaller size of the pane.

The deflection of the pane is calculated as:

$$w_{max} = k_w \cdot \frac{f_{e,d} \cdot a^4}{E \cdot {d_e}^3} = 0{,}075294 \cdot \frac{0{,}32 \cdot 1500^4}{70 \cdot 10^6 \cdot 7^3} = 5{,}08\ mm$$

where w_{max} is deflection;
E modulus of elasticity.

KZ12 assessment:

$$w_{max} = 5{,}08\ mm\ \leq \delta_{lim} = \frac{a}{100} = \frac{1500}{100} = 15\ mm \quad \Rightarrow \quad SATISFACTORY$$

The pane is at SLS **satisfactory** for the applied load.

2.7 Literature

[1] ČSN EN 1991-1-4, *Eurokód 1: Zatížení konstrukcí: Část 1-4: Obecná zatížení – Zatížení větrem*, 2013. Ed. 2. Úřad pro technickou normalizaci, metrologii a státní zkušebnictví

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[3] DIN 18008-2, *Glas im Bauwesen – Bemessungs- und Konstruktionsregeln: Teil 2: Linienförmig gelagerte Verglasungen*, 2011. Berlin: Deutsches Institut für Normung.

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